Math 2E Quiz 3 Afternoon - April 14th

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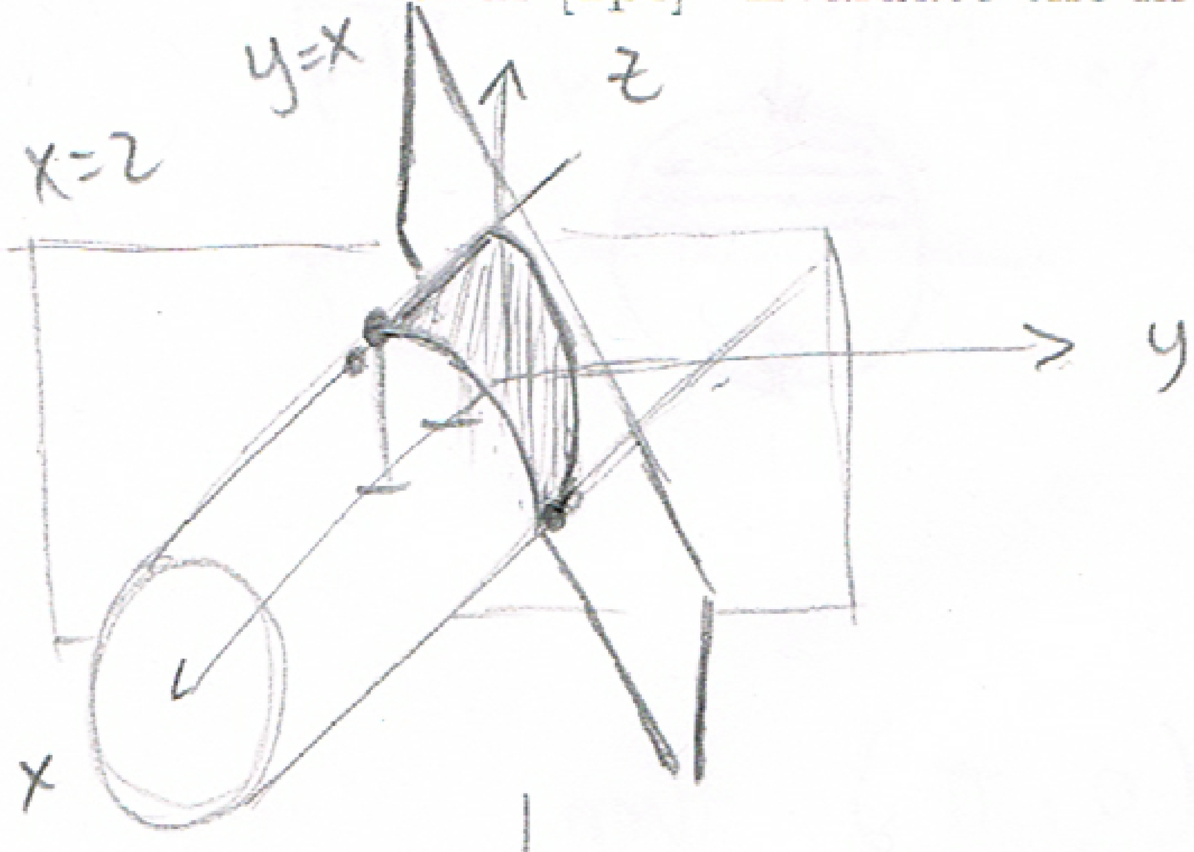
Show all of your work, and simplify all your answers. *There is a question on the back side.

1. Express the volume of the wedge in the first octant that is cut from the cylinder $y^2 + z^2 = 4$ by the planes $y = x$ and $x = 2$. You do not need to evaluate the integral for full credit.

Bonus [1pt] Evaluate the integral to find the volume.

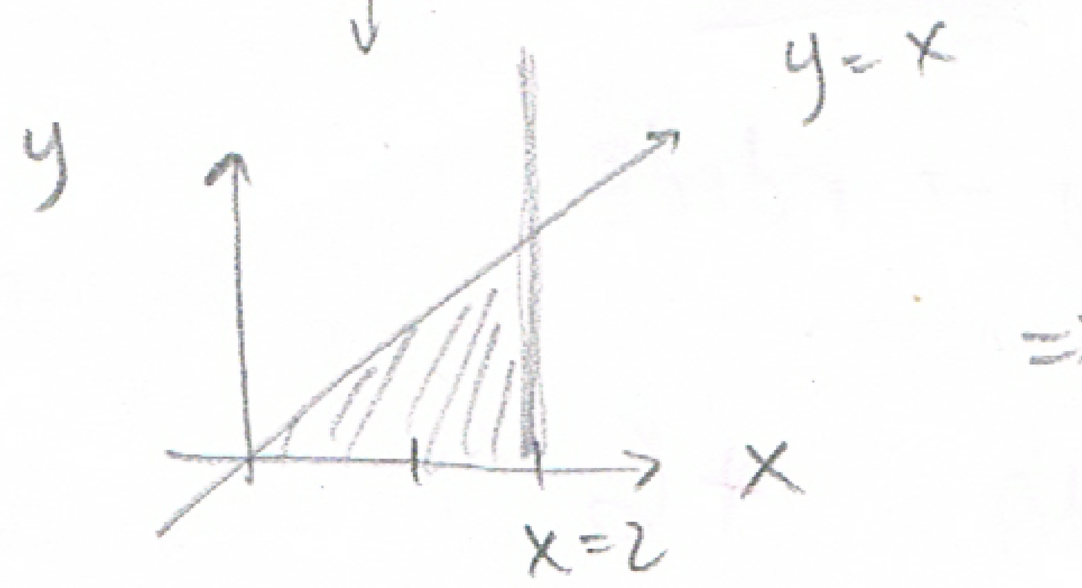


1^{st} octant $\Rightarrow$ $x, y, z > 0$.

Top surface $\Rightarrow$ cylinder $\Rightarrow$ solve for z,

$z = \pm\sqrt{4-y^2}$; Take (+) for top of cylinder, $z > 0$.

+3



+2

$$\Rightarrow \boxed{V = \int_{x=0}^{x=2} \int_{y=0}^{y=x} \int_{z=0}^{z=\sqrt{4-y^2}} 1\, dz\, dy\, dx}$$

+5 (either)

$$\left(\underline{\underline{\text{or}}} \quad \int_{y=0}^{y=2} \int_{x=y}^{x=2} \int_{z=0}^{\sqrt{4-y^2}} 1\, dz\, dy\, dx \right)$$

$\boxed{\text{Compute:}}$ $= \int_{y=0}^{y=2} \int_{x=y}^{x=2} \sqrt{4-y^2}\, dx\, dy$ // constant wrt. x

$$= \int_0^2 \sqrt{4-y^2}\,(2-y)\,dy = \int_0^2 \left(2\sqrt{4-y^2} - y\sqrt{4-y^2}\right) dy$$

① $y = 2\sin\theta$, $dy = 2\cos\theta$

② $u = 4-y^2$; $du = -2y\,dy$

$$= \int_0^{\pi/2} \underbrace{8\cos^2\theta}_{4(1+\cos 2\theta)}\, d\theta - \int_4^0 \sqrt{u}\,\frac{du}{-2}$$

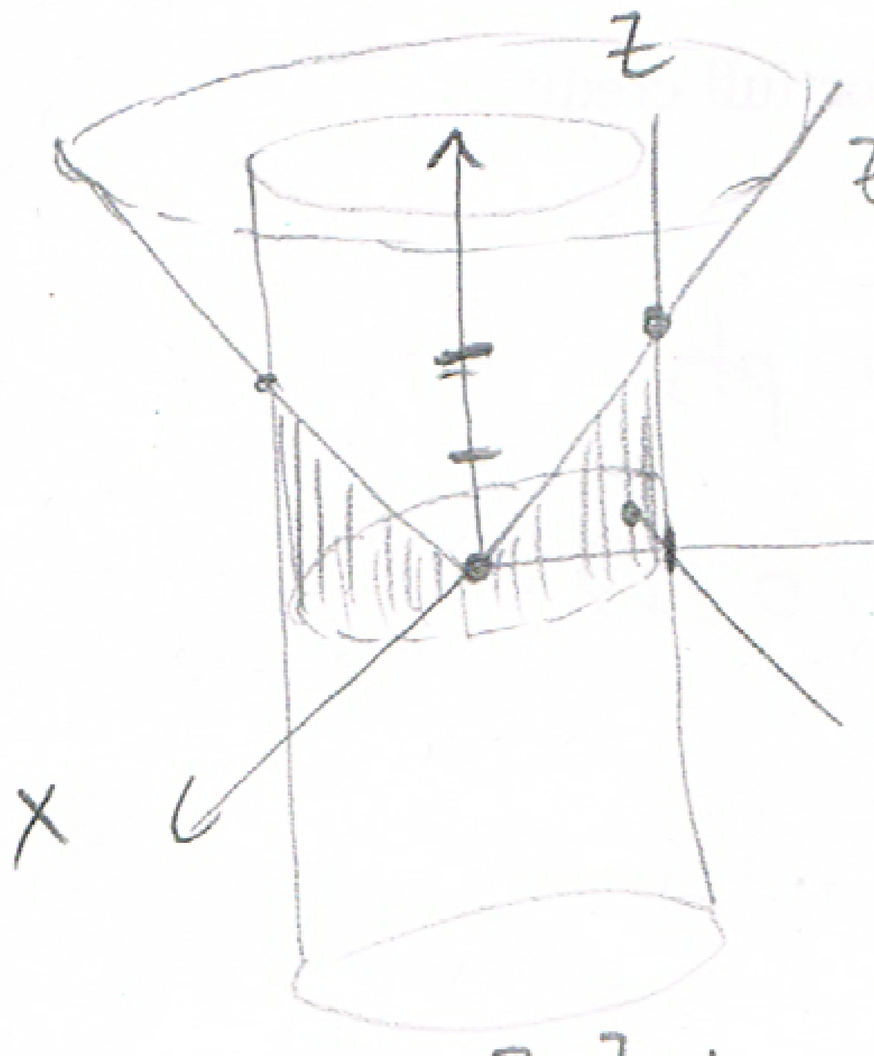
$$= 4\theta + \frac{\sin 2\theta}{2}\Big|_0^{\pi/2} + \frac{1}{2}\cdot\frac{2u^{3/2}}{3}\Big|_4^0$$

$$= \boxed{2\pi - \frac{8}{3}}$$

(+1 Bonus)

2. Let E be the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$. With this region E, evaluate

$$\iiint_E x^2\,dV.$$



$z^2 = 4x^2+4y^2 = \underline{4r^2}$

E is $\underline{\text{under}}$ $\underline{\text{the cone!}}$

$x^2+y^2=1$

(ie $r^2=1$)

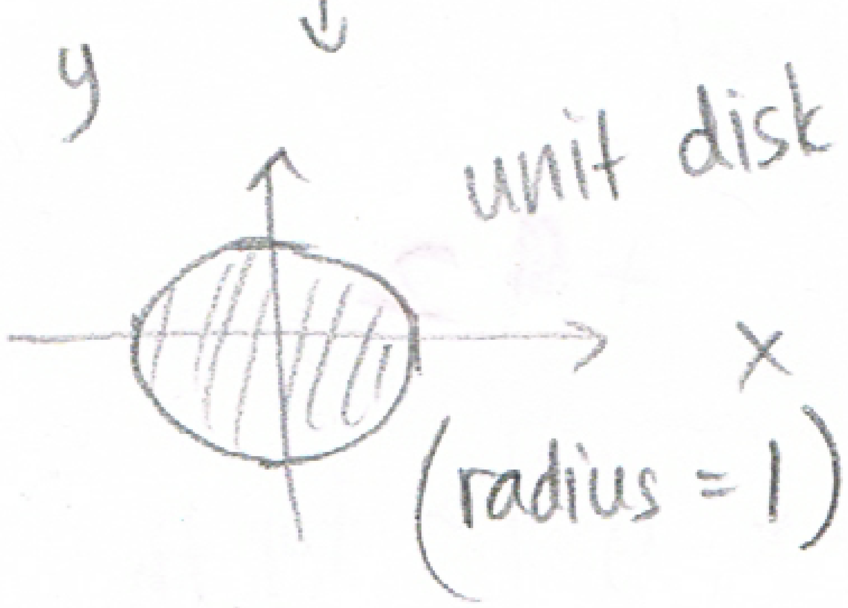
so $\quad \underline{0} \le z \le \sqrt{4x^2+y^2} = \sqrt{4r^2} = \underline{2r}$

and the xy-domain is a disc.

$\therefore \iiint_E x^2\,dV$ equals

+5 $= \displaystyle\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1}\int_{z=0}^{z=2r} r^2\cos^2\theta \cdot \underline{r\,dz\,dr\,d\theta}.$

$= \displaystyle\int_0^{2\pi}\int_0^1 r^3\cos^2\theta\cdot(2r-0)\,dr\,d\theta$



unit disk

(radius = 1)

$= \displaystyle\int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\,dr\,d\theta$

$= \displaystyle\int_0^{2\pi} \frac{2r^5}{5}\cos^2\theta\,\Big|_0^1\,d\theta = \int_0^{2\pi}\frac{2}{5}\cos^2\theta\,d\theta$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ ID

$= \displaystyle\int_0^{2\pi} \frac{\cancel{2}}{5}\cdot\frac{1}{\cancel{2}}(1+\cos 2\theta)\,d\theta$ +3

$= \frac{1}{5}\left[\theta + \frac{\sin 2\theta}{2}\right]\Big|_0^{2\pi} = \boxed{\frac{2\pi}{5}}$ +2